Introduction to QCD on a Lattice

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Path Integral

Field Theory

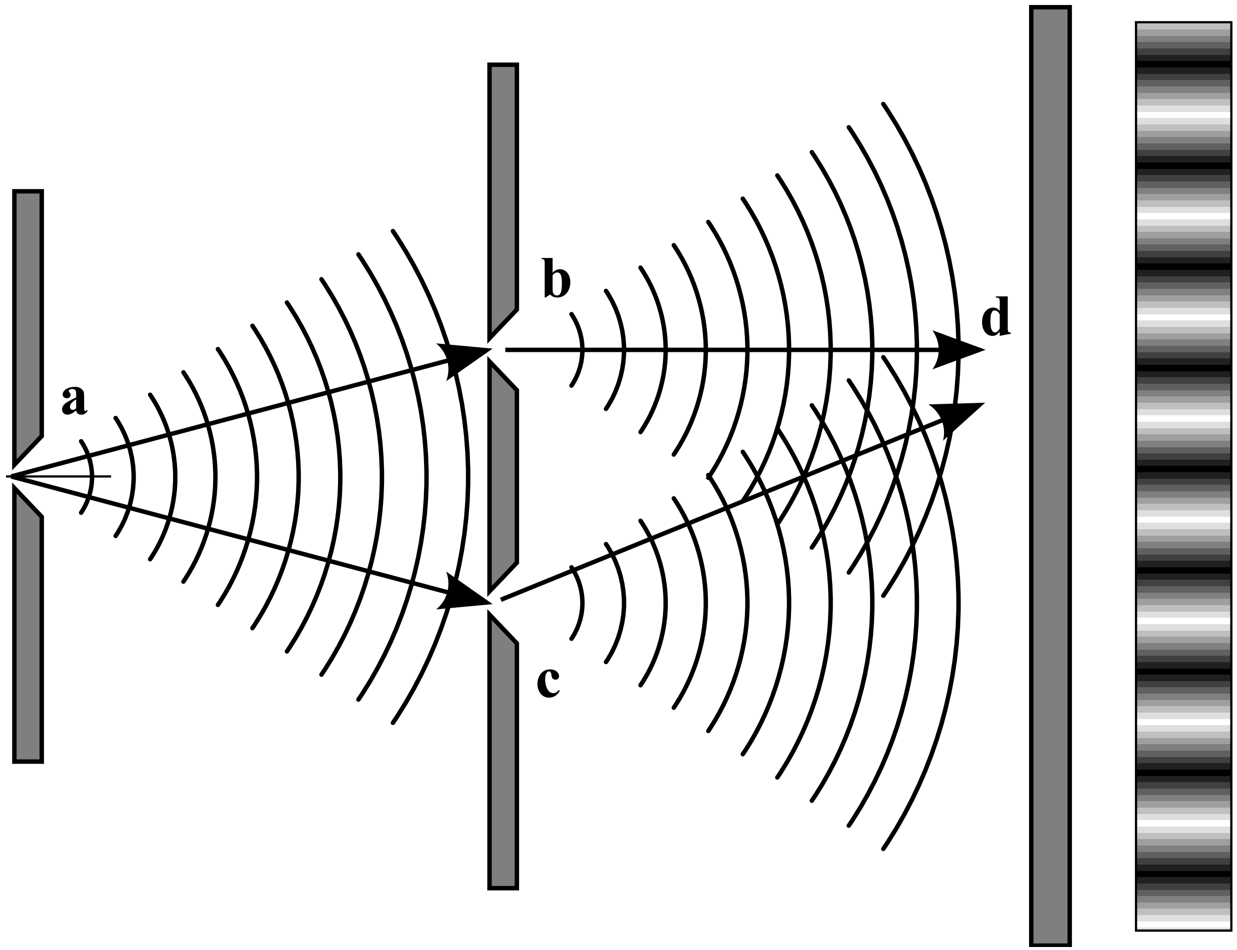
Lattice QCD

Temperature & Density

QCD

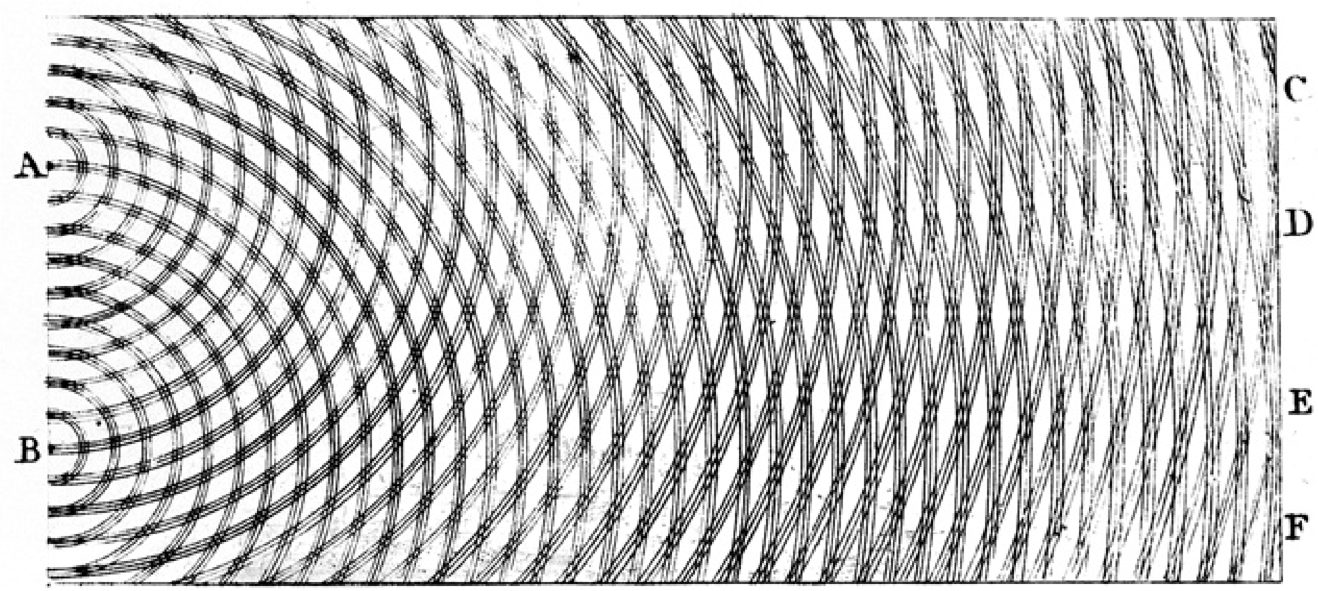
- QUANTUM: Basically a quantum field theory.
- CHROMO: Fancy name for COLOR. A property similar to ELECTRO.
- DYNAMICS: How the fields interacting with each other. Quarks interact with each other by exchanging gluons.
- QUARK: like electron.
- GLUON: like photon.

Wave-Particle Duality



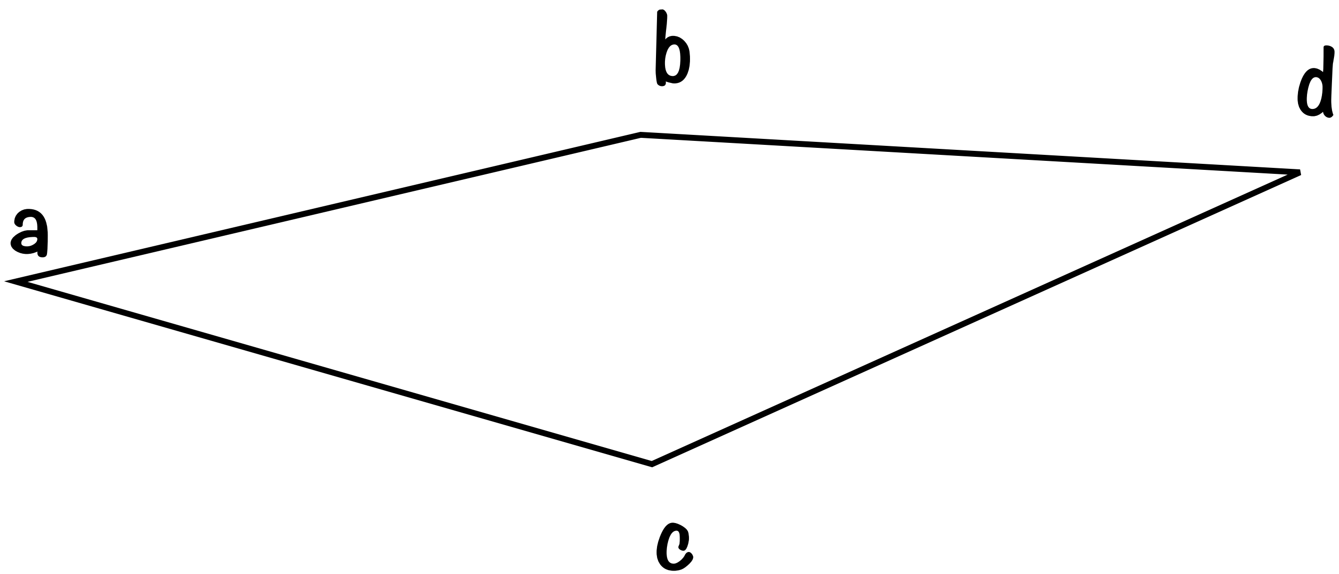
Young's double-slit experiment.

Wave-Particle Duality



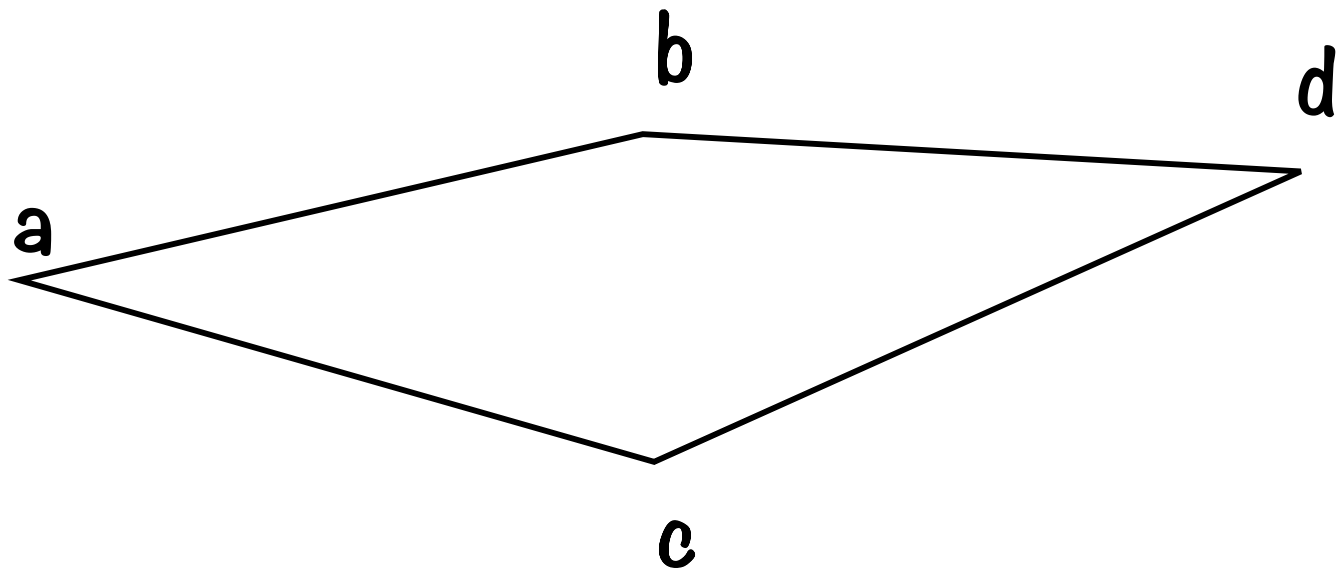
Thomas Young's sketch of two-slit diffraction of waves, 1803.

Double Slits



$$
\begin{aligned}
A(a \longrightarrow d) &= A(a \longrightarrow b \longrightarrow d) \\
&\quad + A(a \longrightarrow c \longrightarrow d) \\
&= A(a \longrightarrow b)A(b \longrightarrow d) \\
&\quad + A(a \longrightarrow c)A(c \longrightarrow d)
\end{aligned}
$$

Double Slits



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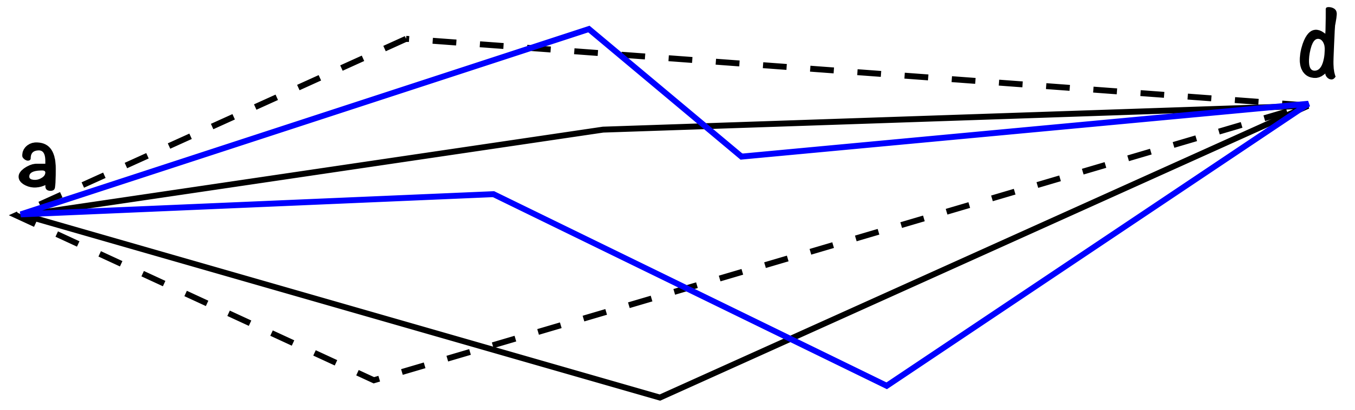
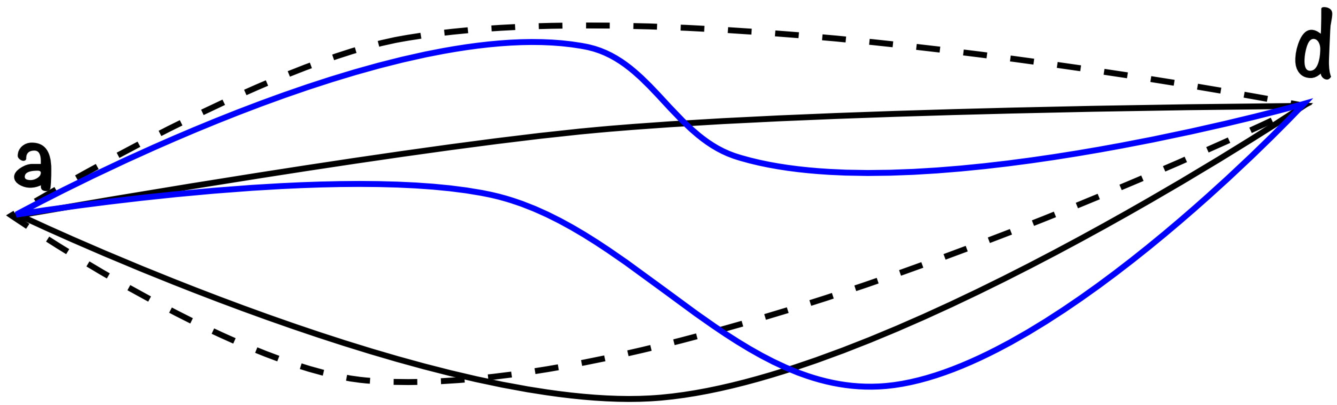
Recursion!

Infinite Slits



$$A(a \to d) = \sum_{\text{paths}} \prod_{\text{piece} \in \text{path}} A(\text{piece})$$

No Slits



$$A(a \rightarrow d) = \sum_{\text{paths}} \prod_{\text{piece} \in \text{path}} A(\text{piece})$$

- Path from point a to d is fixed from time T_a to T_d.
- Think of a and d are two points on a space-time lattice.

Quantum Field Theory

- Instead of one point moving through space-time, the field lives at all the space-time points.

- Instead of summing over all the space-time paths, the field is integrated over all the possible values at all the space-time points.

$$\prod_{x,t} \int d\phi_{x,t}\, e^{-S(\phi)} \equiv \int D\phi\, e^{-S(\phi)}$$

Quantum Chromodynamics

$$Z = \prod_{x,\mu} \int dU_{x,\mu}\, d\bar{\Psi}_x\, d\Psi_x\, e^{-S(U) - \bar{\Psi} M(U) \Psi}$$

$$\equiv \int DU\, e^{-S(U)} \det[M(U)]$$

$$\int 1\, d\Psi = 0$$

$$\int \Psi\, d\Psi = 1$$

$$\int e^{\bar{\Psi} M \Psi}\, d\bar{\Psi}\, d\Psi = \det[M]$$

§ Ψ describes the quark field, and lives on every point of the space-time lattice.

§ 3×3 complex matrices, U, live on every link on the space-time lattice from x to $x+\mu$.

§ $M(U)$ is a HUGE matrix of $L\times L$, where $L=12\times\text{Volume}$.

What do we want?

$$\langle O \rangle = \frac{\int DU\, O(U, M^{-1}) e^{-S(U)} \det[M(U)]}{Z}$$

- O, usually called observable (or operator), is what we ultimately want to calculate.
- It is a multidimensional integral with a dimension of the lattice volume.
- Use Markov chain Monte Carlo.

Markov Chain Monte Carlo

$$\langle O \rangle = \frac{\int DU\, O(U, M^{-1}) e^{-S(U)} \det[M(U)]}{Z}$$

Generate U:

$$\mathrm{Prob}[U] \propto \det[M] e^{-S}$$

with N numbers of U:

$$\langle O \rangle \simeq \frac{1}{N} \sum_{k=1}^{N} O(U^{(k)})$$

Finite Temperature & Density

- All above is actually zero temperature and zero density — the VACUUM.
- Vacuum is NOT nothing!
- Finite temperature & density is more!
- FINITE: neither zero, nor high.
- Temperature is around 100 MeV, which is 10^{12} °C.
- There is phase transition when temperature and density is increased.

Finite Temperature

- On the lattice, space-time is periodic (for gauge U).
- Our functional integral becomes a trace.
- Temperature, T, is simply the inverse imaginary time, $1/\tau$.
- Finite τ gives finite T; large τ, low T.

$$\langle O \rangle = \frac{\mathrm{Tr}\,(O e^{-H\tau})}{\mathrm{Tr}\, e^{-H\tau}} = \frac{\mathrm{Tr}\,(O e^{-H/T})}{\mathrm{Tr}\, e^{-H/T}}$$

Finite Density

$$\langle O \rangle = \frac{\mathrm{Tr}\,(O e^{-(H-\mu B)/T})}{\mathrm{Tr}\, e^{-(H-\mu B)/T}}$$

- § μ is the chemical potential.
- § B is the number that represents the density.
- § Unfortunately, our determinant is complex on the lattice.

$$\mathrm{Prob}[U] \propto \det[M] e^{-S}, \quad \det[M] \not> 0$$

Sign Problem

THANKS