

# Construction of Predictive Neuron Models using Large Scale Data Assimilation

Alain Nogaret

*Department of Physics, University of Bath, UK*

[A.R.Nogaret@bath.ac.uk](mailto:A.R.Nogaret@bath.ac.uk)

## Co-Workers:

Bath: Ashok Chauhan, Joe Taylor, Edward Lander, Paul Curzons, John T Taylor

UCSD: Henry Abarbanel, Mark Kostuk

Chicago: Dan Meliza (now at Univ. Virginia), Dan Margoliash, Hao Huang

Bristol: Julian Paton, Erin O'Callaghan, Fiona McBryde, Davi Moraes

NHS hosp: Edward Duncan MD, Angus Nightingale MD, Dr Guillaume Chanoit MRCVS

Auckland: Rohit Ramchandra MD, Ian LeGrice, Nigel Lever MD



## Building neuron models with data assimilation:

- Multichannel conductance model
- Assimilation of electrophysiological data
- Model validation through prediction
- Bayesian inference boost – single valued model

## DA for conditioning neural networks:

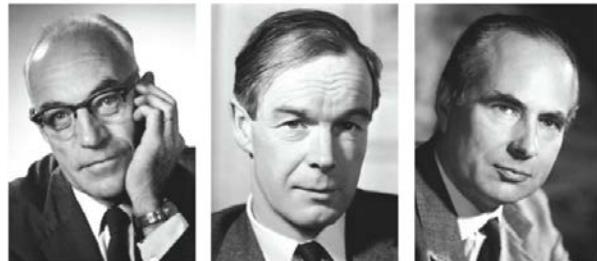
- Adaptive Bioelectronics  
⇒ *New therapies for cardiorespiratory disease*
- Control of switching between chaotic attractors to produce specific motor patterns e.g. gaits  
⇒ *Validation of the command neuron hypothesis*

# Neuron: a current driven non-linear oscillator



The Nobel Prize in Physiology or Medicine 1963

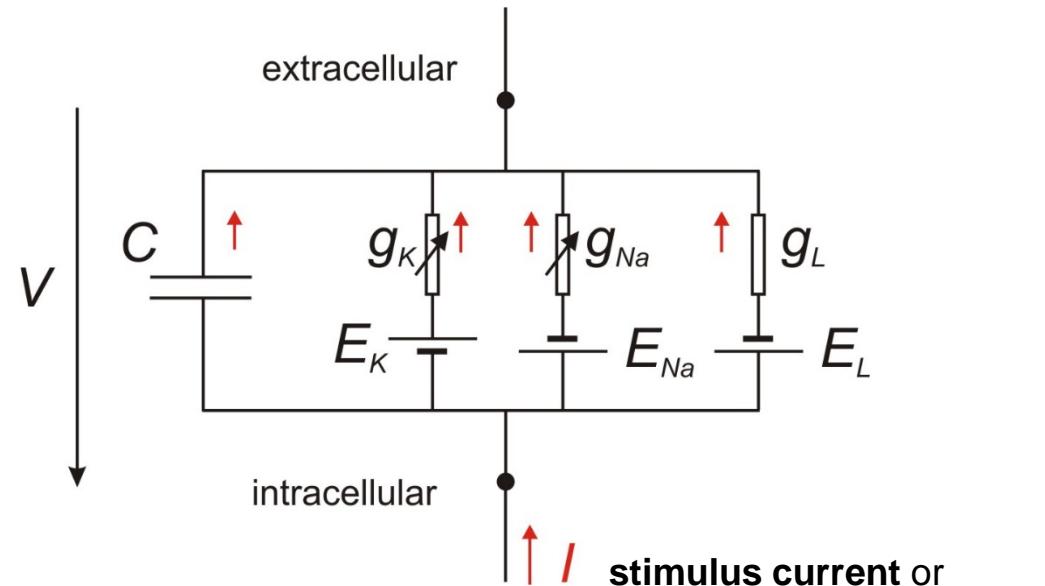
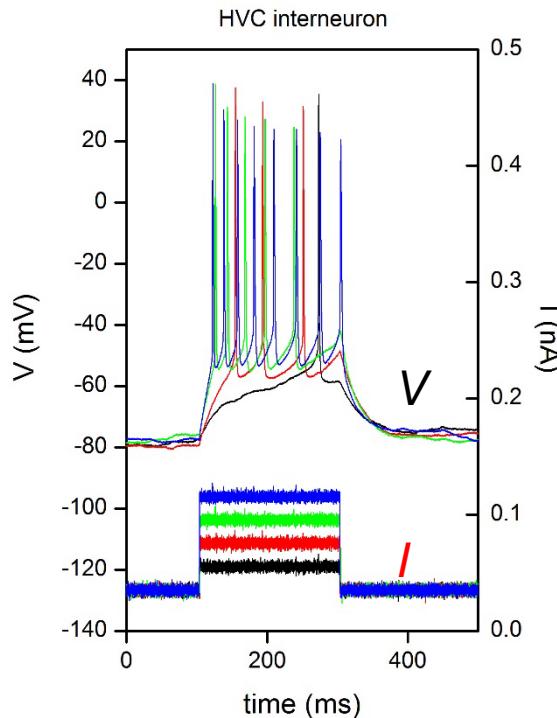
"for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the peripheral and central portions of the nerve cell membrane"



Sir John Carew Eccles

Alan Lloyd Hodgkin

Andrew Fielding Huxley



/ stimulus current or synaptic current injected through the membrane.

**Hodgkin-Huxley model:**

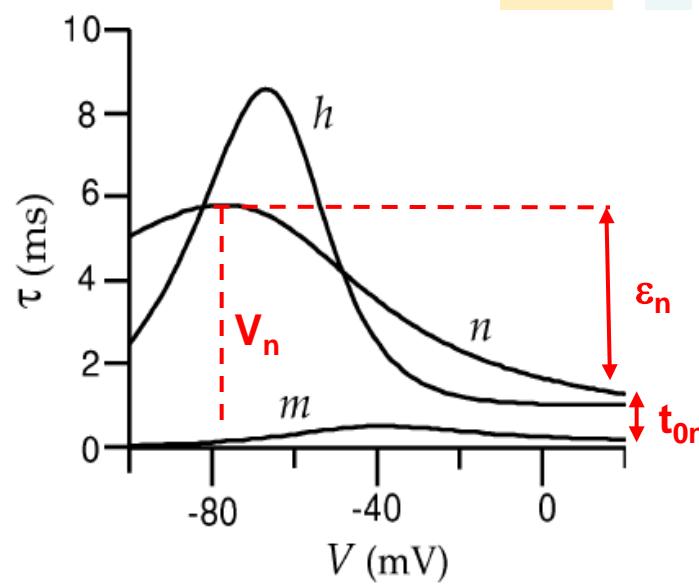
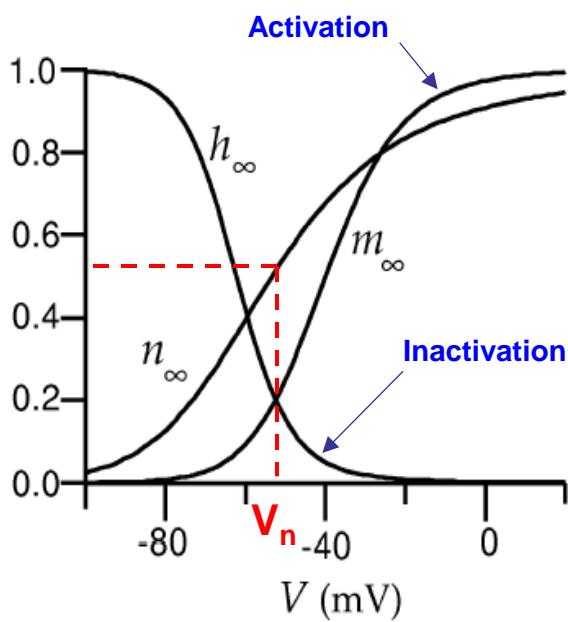
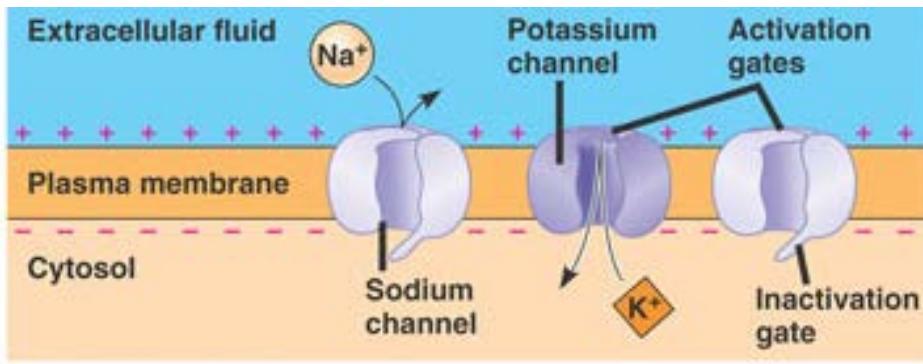
$$C\dot{V} = \bar{g}_{Na}m^3h(E_K - V) + \bar{g}_K n^4(E_{Na} - V) + g_L(E_L - V) + I$$

$$\tau_m \dot{m} = m_\infty - m(t)$$

$$\tau_h \dot{h} = h_\infty - h(t)$$

$$\tau_n \dot{n} = n_\infty - n(t)$$

# Kinetics of ionic gates



**Dynamics of the Potassium-activation gate:**

$$\frac{dn(t)}{dt} = \frac{n_\infty(V) - n(t)}{\tau_n(V)}$$

$$n_\infty(V) = \frac{1}{2} \left[ 1 + \tanh\left(\frac{V - V_n}{dV_n}\right) \right]$$

$$\tau_n(V) = t_{0n} + \varepsilon_n \left[ 1 - \tanh^2\left(\frac{V - V_n}{dV_{nt}}\right) \right]$$

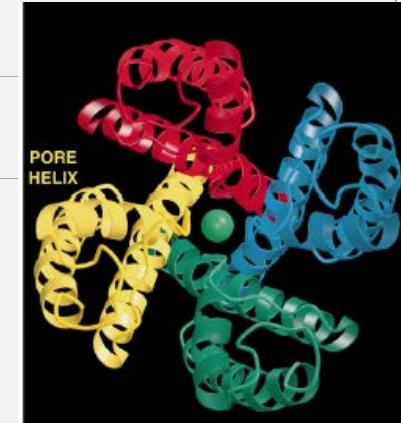
5 model parameters

**Sodium activation and inactivation gates:**

10 model parameters

# Ion channels are highly specialized proteins

Function	Protein name ( <a href="#">associated gene</a> )
<b>Delayed rectifier</b> Slowly activating or non-inactivating	K <sub>v</sub> α1.x - Shaker-related: K <sub>v</sub> 1.1 ( <a href="#">KCNA1</a> ), K <sub>v</sub> 1.2 ( <a href="#">KCNA2</a> ), K <sub>v</sub> 1.3 ( <a href="#">KCNA3</a> ), K <sub>v</sub> 1.5 ( <a href="#">KCNA5</a> ), K <sub>v</sub> 1.6 ( <a href="#">KCNA6</a> ), K <sub>v</sub> 1.7 ( <a href="#">KCNA7</a> ), K <sub>v</sub> 1.8 ( <a href="#">KCNA10</a> ) K <sub>v</sub> α2.x - Shab-related: K <sub>v</sub> 2.1 ( <a href="#">KCNB1</a> ), K <sub>v</sub> 2.2 ( <a href="#">KCNB2</a> ) K <sub>v</sub> α3.x - Shaw-related: K <sub>v</sub> 3.1 ( <a href="#">KCNC1</a> ), K <sub>v</sub> 3.2 ( <a href="#">KCNC2</a> ) K <sub>v</sub> α7.x: K <sub>v</sub> 7.1 ( <a href="#">KCNQ1</a> ) - <a href="#">KvLQT1</a> , K <sub>v</sub> 7.2 ( <a href="#">KCNQ2</a> ), K <sub>v</sub> 7.3 ( <a href="#">KCNQ3</a> ), K <sub>v</sub> 7.4 ( <a href="#">KCNQ4</a> ), K <sub>v</sub> 7.5 ( <a href="#">KCNQ5</a> ) K <sub>v</sub> α10.x: K <sub>v</sub> 10.1 ( <a href="#">KCNH1</a> )
<b>A-type potassium channel</b> rapidly inactivating	K <sub>v</sub> α1.x - Shaker-related: K <sub>v</sub> 1.4 ( <a href="#">KCNA4</a> ) K <sub>v</sub> α3.x - Shaw-related: K <sub>v</sub> 3.3 ( <a href="#">KCNC3</a> ), K <sub>v</sub> 3.4 ( <a href="#">KCNC4</a> ) K <sub>v</sub> α4.x - Shal-related: K <sub>v</sub> 4.1 ( <a href="#">KCND1</a> ), K <sub>v</sub> 4.2 ( <a href="#">KCND2</a> ), K <sub>v</sub> 4.3 ( <a href="#">KCND3</a> )
<b>Outward rectifying</b> passes current more easily outwards	K <sub>v</sub> α10.x: K <sub>v</sub> 10.2 ( <a href="#">KCNH5</a> )
<b>Inward rectifying</b> passes current more easily inwards	K <sub>v</sub> α11.x: K <sub>v</sub> 11.1 ( <a href="#">KCNH2</a> ), K <sub>v</sub> 11.2 ( <a href="#">KCNH6</a> ), K <sub>v</sub> 12.3 ( <a href="#">KCNH7</a> )
<b>Slowly activating</b>	K <sub>v</sub> α12.x: K <sub>v</sub> 12.1 ( <a href="#">KCNH8</a> ), K <sub>v</sub> 12.2 ( <a href="#">KCNH3</a> ), K <sub>v</sub> 12.3 ( <a href="#">KCNH4</a> )
<b>Modifier / silencer</b> heterotetramerize with K <sub>v</sub> α2 family members to form conductive channels	K <sub>v</sub> α5.x: K <sub>v</sub> 5.1 ( <a href="#">KCNF1</a> ) K <sub>v</sub> α6.x: K <sub>v</sub> 6.1 ( <a href="#">KCNG1</a> ), K <sub>v</sub> 6.2 ( <a href="#">KCNG2</a> ), K <sub>v</sub> 6.3 ( <a href="#">KCNG3</a> ) K <sub>v</sub> 6.4 ( <a href="#">KCNG4</a> ) K <sub>v</sub> α8.x: K <sub>v</sub> 8.1 ( <a href="#">KCNV1</a> ), K <sub>v</sub> 8.2 ( <a href="#">KCNV2</a> ) K <sub>v</sub> α9.x: K <sub>v</sub> 9.1 ( <a href="#">KCNS1</a> ), K <sub>v</sub> 9.2 ( <a href="#">KCNS2</a> ), K <sub>v</sub> 9.3 ( <a href="#">KCNS3</a> )



Individual ion channels have specific threshold voltages and kinetics.

# The 9 ion channel conductance model

$$C_m \frac{dV}{dt} = -J_{ion} + \frac{I_{app} + V / R_s}{area}$$

where:

$$-J_{ion} = J_{NaT} + J_{NaP} + J_{K1} + J_{K2} + J_{K3} + J_{CaL} + J_{CaT} + J_{HCN} + J_L$$

and  $C_m = C / area$  membrane capacitance per unit area.

$J_{ion} = I_{ion} / area$  ion current density

ID	Channel	Current density	Nominal conductance
NaT	Fast and transient Na <sup>+</sup> current	$J_{NaT} = g_{NaT} m^3 h(E_{Na} - V)$	$g_{NaT} = 110 \text{mS.cm}^{-2}$
NaP	Persistent Na <sup>+</sup> current	$J_{NaP} = g_{NaP} m(E_{Na} - V)$	$g_{NaP} = 0.064 \text{mS.cm}^{-2}$
K1	Transient depolarization activated K <sup>+</sup> current	$J_{K1} = g_{K1} m^4 (E_K - V)$	$g_{K1} = 5 \text{mS.cm}^{-2}$
K2	Rapidly inactivating K <sup>+</sup> current (A current)	$J_{K2} = g_{K2} m^4 h(E_K - V)$	$g_{K2} = 12 \text{mS.cm}^{-2}$
K3	Ca <sup>2+</sup> activated K <sup>+</sup> current	$J_{K3} = g_{K3} m(E_K - V)$	$g_{K3} = 9.1 \text{mS.cm}^{-2}$
CaL	High threshold Ca <sup>2+</sup> current	$J_{CaL} = \rho m^2 J_{Ca}$	-
CaT	Low threshold Ca <sup>2+</sup> current	$J_{CaT} = m^2 h J_{Ca}$	-
HCN	Hyperpolarization-activated cation current	$J_{HCN} = g_{HCN} h(E_{HCN} - V)$	$g_{HCN} = 0.092 \text{mS.cm}^{-2}$
Leak	Leakage channels (K & Na)	$J_L = g_L (E_L - V)$	$g_L = 0.066 \text{mS.cm}^{-2}$

**Calcium current prefactor:**  $J_{Ca} = \left( \frac{g_{out} - g_{in} \exp(V/V_T)}{\exp(V/V_T) - 1} \right) \times V$  Goldman-Hodgkin-Katz equation

12 state variables, 71 parameters  
12 coupled non linear differential equations

# Model equations

- Voltage :**  $dy_1/dt = ((p_2y_2^3y_3 + p_3y_4)(p_4 - y_1) + (p_5y_5^4 + p_6y_6^4y_7 + p_7y_8)(p_8 - y_1)$
- $+ (p_{71}y_9^2 + p_{72}y_{10}^2y_{11})19.2970673(p_{11} - 0.0001\exp(y_1/13))/\text{GHK}(y_1)$
- $+ p_9(p_{10} - y_1) + p_{12}y_{12}(-43 - y_1) + I_{inj}/p_{13})/p_1 + u(t)(V_{data}(t) - y_1)$
- NaT, m :**  $dy_2/dt = 0.5(1 + \tanh((y_1 - p_{14})/p_{15}) - 2y_2)/(p_{17} + p_{18}(1 - \tanh^2((y_1 - p_{14})/p_{16})))$
- NaT, h :**  $dy_3/dt = 0.5(1 + \tanh((y_1 - p_{19})/p_{20}) - 2y_3)/(p_{22} + p_{23}(1 - \tanh^2((y_1 - p_{19})/p_{21})))$
- NaP, m :**  $dy_4/dt = 0.5(1 + \tanh((y_1 - p_{24})/p_{25}) - 2y_4)/(p_{27} + p_{28}(1 - \tanh^2((y_1 - p_{24})/p_{26})))$
- K1, m :**  $dy_5/dt = 0.5(1 + \tanh((y_1 - p_{29})/p_{30}) - 2y_5)/(p_{32} + p_{33}(1 - \tanh^2((y_1 - p_{29})/p_{31})))$
- K2, m :**  $dy_6/dt = 0.5(1 + \tanh((y_1 - p_{34})/p_{35}) - 2y_6)/(p_{37} + p_{38}(1 - \tanh^2((y_1 - p_{34})/p_{36})))$
- K2, h :**  $dy_7/dt = 0.5(1 + \tanh((y_1 - p_{39})/p_{40}) - 2y_7)/(p_{42} + p_{44} + 0.5(1 - \tanh(y_1 - p_{39}))$
- $\cdot (p_{43}(1 - \tanh^2((y_1 - p_{39})/p_{41})) - p_{44}))$
- K3, m :**  $dy_8/dt = 0.5(1 + \tanh((y_1 - p_{45})/p_{46}) - 2y_8)/(p_{48} + p_{49}(1 - \tanh^2((y_1 - p_{45})/p_{47})))$
- CaT, m :**  $dy_9/dt = 0.5(1 + \tanh((y_1 - p_{50})/p_{51}) - 2y_9)/(p_{53} + p_{54}(1 - \tanh^2((y_1 - p_{50})/p_{52})))$
- CaL, m :**  $dy_{10}/dt = 0.5(1 + \tanh((y_1 - p_{55})/p_{56}) - 2y_{10})/(p_{58} + p_{59}(1 - \tanh^2((y_1 - p_{55})/p_{57})))$
- CaL, h :**  $dy_{11}/dt = 0.5(1 + \tanh((y_1 - p_{60})/p_{61}) - 2y_{11})/(p_{64} + p_{65}(1 + \tanh((y_1 - p_{60})/p_{62}))$
- $\cdot (1 - \tanh((y_1 - p_{60})/p_{63}))(1 - \tanh(y_1 - p_{60}) \tanh((1/p_{62} + 1/p_{63})(y_1 - p_{60})))$
- $/(1 + \tanh((y_1 - p_{60})/p_{62}) \tanh((y_1 - p_{60})/p_{63})))$
- HCN, h :**  $dy_{12}/dt = 0.5(1 + \tanh((y_1 - p_{66})/p_{67}) - 2y_{12})/(p_{69} + p_{70}(1 - \tanh^2((y_1 - p_{66})/p_{68})))$

**Takens embedding theorem:** all information required to constrain the model is contained in the observation of one state variable – the membrane voltage  $V(t)$  – over a finite time window of duration  $T$ .

**Record time series data to be fitted:**  $V(t_1), V(t_2), V(t_3), \dots V(t_N)$  induced by the current protocol injected in the neuron  $I(t_1), I(t_2), I(t_3), \dots I(t_N)$ .

**Define a cost function to minimize:**

$$C(y_1(0), \vec{p}) = \frac{1}{2} \sum_{i=0}^N \left\{ (V(t_i) - y_1(t_i, \vec{p}))^2 + u(t_i)^2 \right\}$$

↑                      ↑                      ↑  
 Observed            Model            Convergence  
 voltage             variable        acceleration function      Creveling et al, Phys. Lett. A,  
 Creveling et al, Phys. Lett. A,  
 372, 2640 (2008)

under:

- **Equality constraints:** the 12 differential equations linearized at each point of the discretized time window  $t=iT/N$ . Typically  $N=100,000$  mesh points  $\Rightarrow$  1.2 million constraints
- **Inequality constraints:** the search intervals of the 72 parameters

**Build the Lagrangian** by replacing the inequality constraints with logarithmic barriers  
 $\Rightarrow$  Karush-Kuhn-Tucker system: sparse systems of linear equations

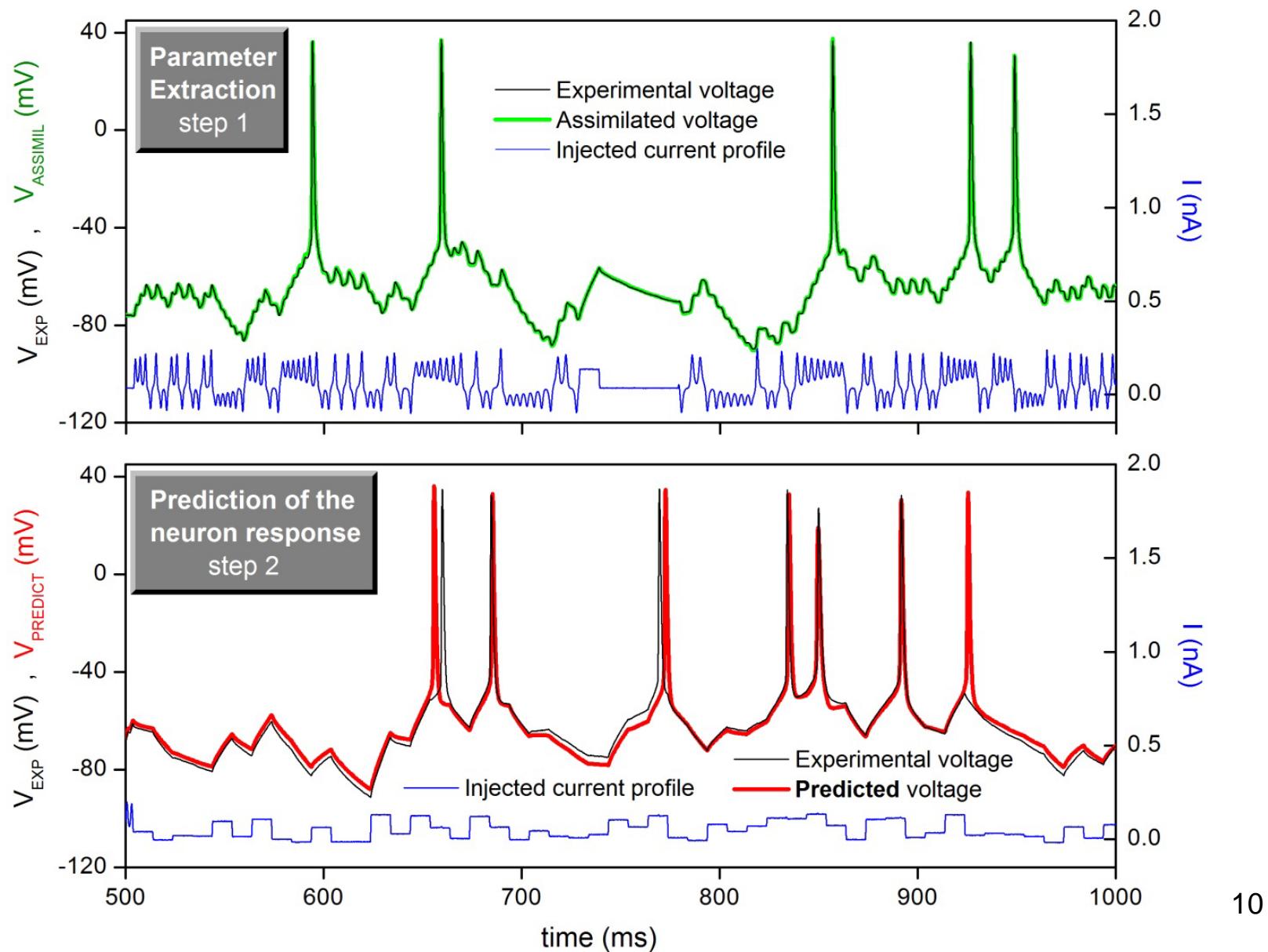
**Interior point optimization:** iteratively seek the extremum of the KKT Lagrangian using Newton's method.  
At each iteration reduce the height of the logarithmic barrier until convergence is achieved.

**When data assimilation has converged** ~ 2 days of run time on a workstation:

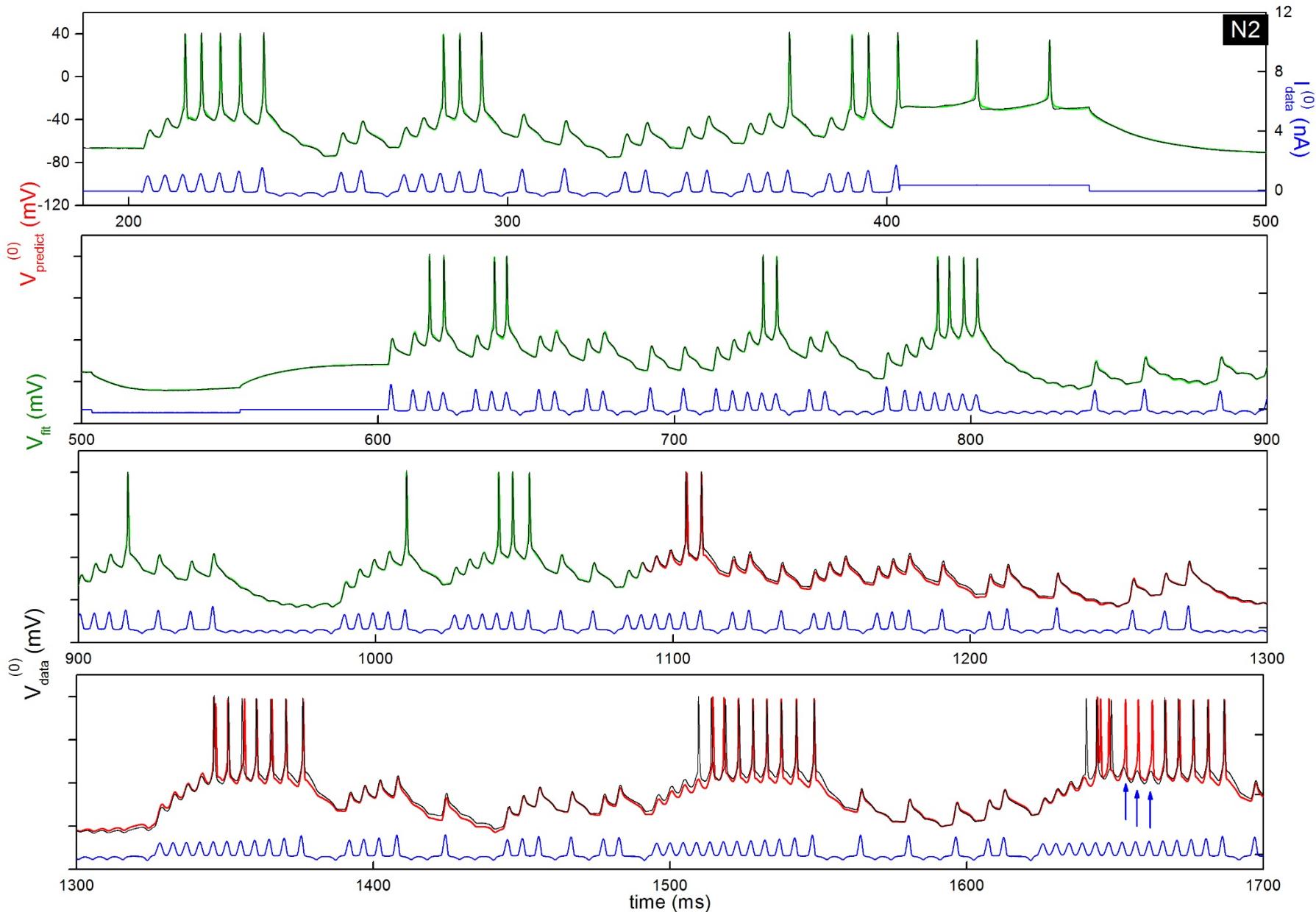
- The  $\vec{p}=\{C_m, g_{Na}, E_{Na}, g_K, E_K \dots\}$  vector solution of the problem is obtained – 71 parameters.
- $u(t)=0$

## Example of extracted parameter sets

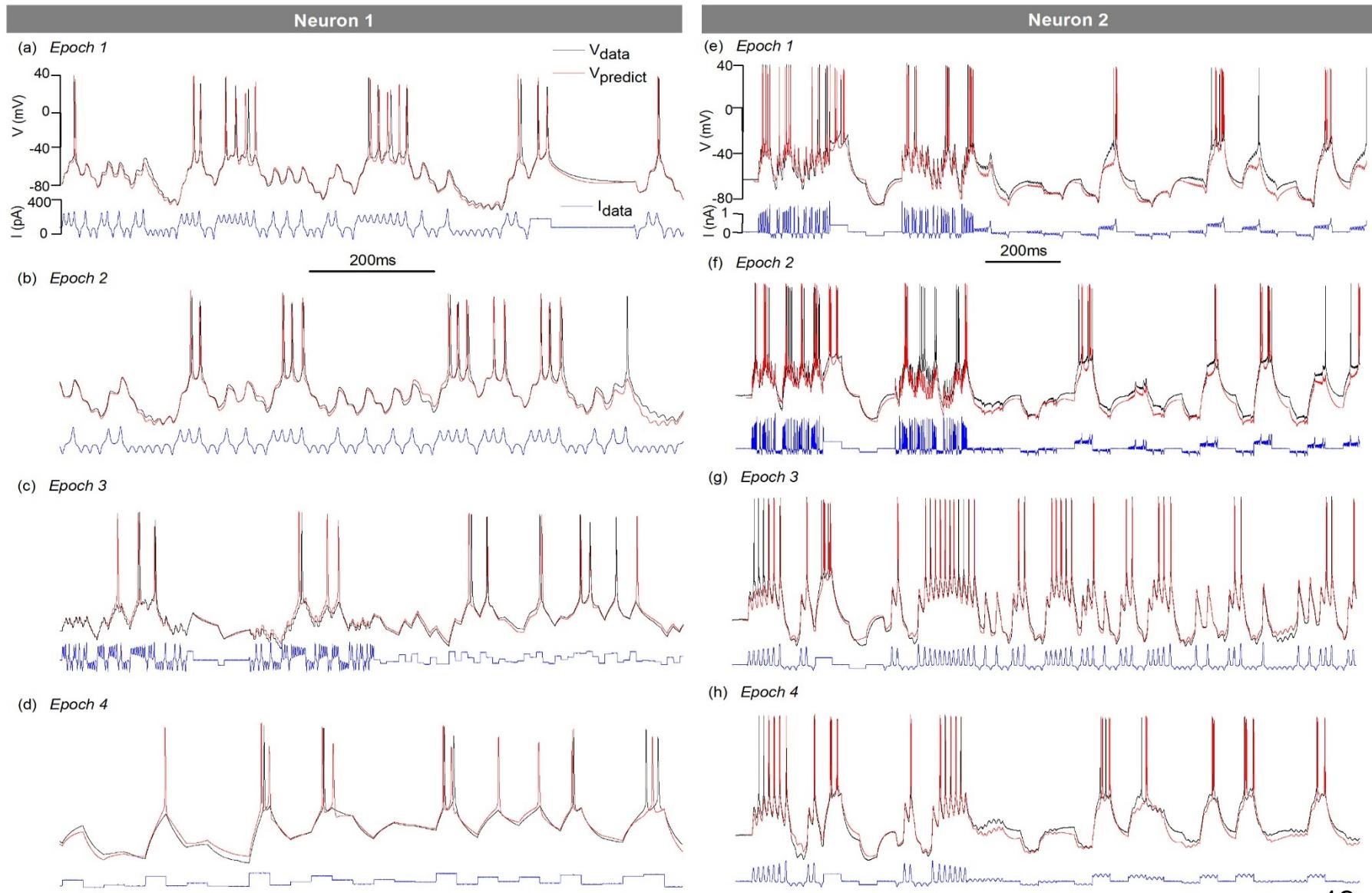
# Assimilation and prediction of neuron output – N1



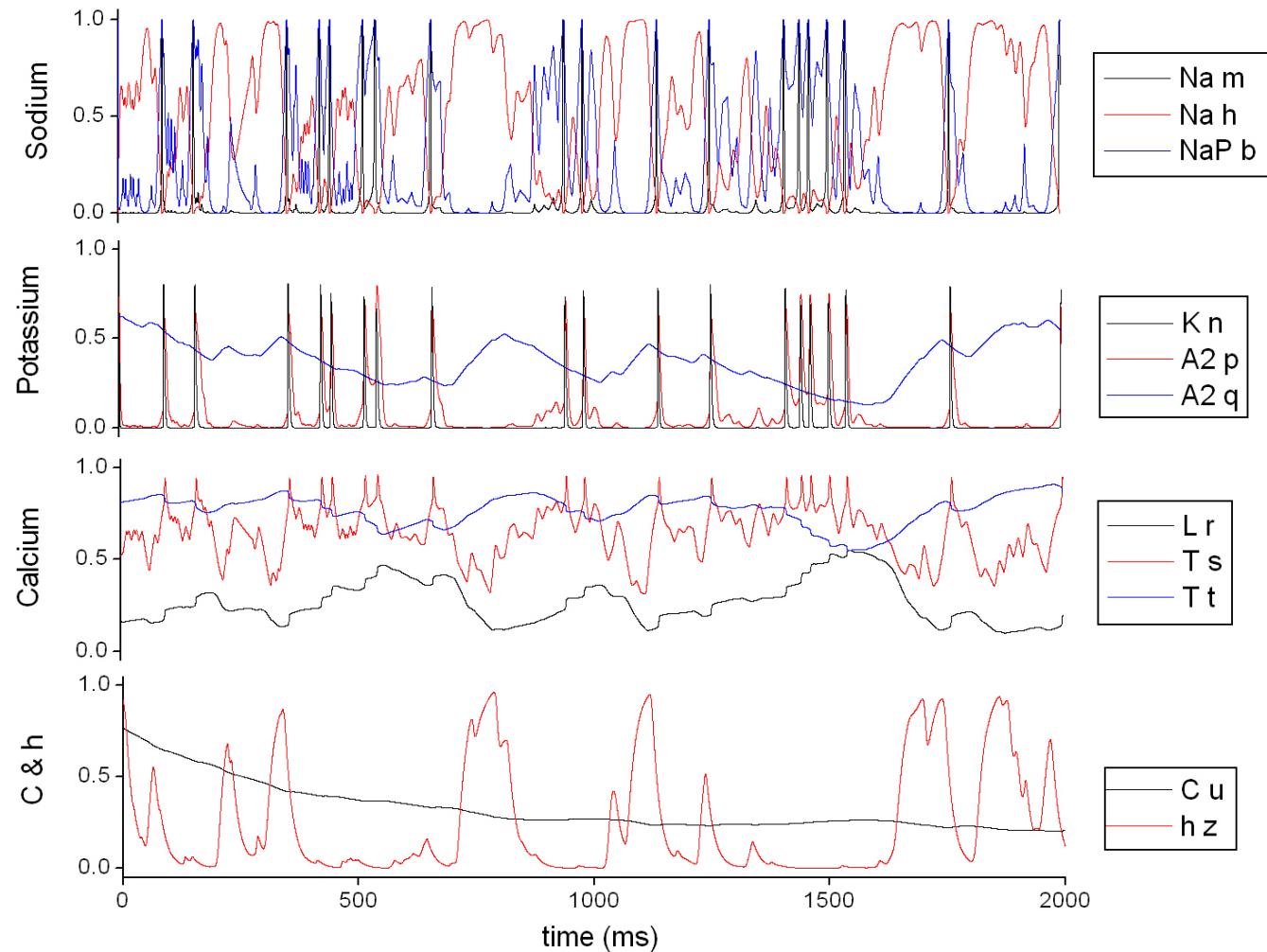
# Assimilation and prediction of neuron output – N2



# Prediction neuron response to arbitrary current stimulation



# Prediction of the state of ionic gates



Predicting the state of the gate variables of the 9 ion channels – **not accessible to the experiment!**

# Uncertainty on parameter field

Separate the experimental signal  $V_{data}(t)$  into the useful signal  $V_{use}(t)$  and the noise component  $v_n(t)$ .

Insert in the cost function:

$$\begin{aligned}
 c(\vec{x}(0), \vec{p}) &= \frac{1}{2} \sum_{i=0}^{i=N} (V_{data}(t_i) - V(t_i, \vec{x}(0), \vec{p}))^2 \\
 &= \frac{1}{2} \sum_{i=0}^{i=N} (V_{use}(t_i) + v_n(t_i) - V(t_i, \vec{x}(0), \vec{p}))^2 \\
 &= \frac{1}{2} \sum_{i=0}^{i=N} (V_{use}(t_i) - V(t_i, \vec{x}(0), \vec{p}))^2 + \frac{1}{2} \sum_{i=0}^{i=N} (v_n(t_i))^2
 \end{aligned}$$

The cross term cancels as it is proportional to the noise average which is zero. The second term on the RHS gives the variance of the membrane voltage which when driven by thermal fluctuations is given by Nyquist theorem:

$$c(\vec{x}(0), \vec{p}) = \frac{1}{N+1} \sum_{i=0}^{i=N} (v_n(t_i))^2 = 4Rk_B T \Delta f$$

Entropy  $T$   
 $c(\vec{x}(0), \vec{p}) = \frac{1}{2} \sum_{i=0}^{i=N} (V_{use}(t_i) - V(t_i, \vec{x}(0), \vec{p}))^2 + 2(N+1)Rk_B \Delta f$   
 Total energy: Free energy: Random energy:  
 $U$   $F$   $TS$   
F=0 achieved by direct parameter search T=0 achieved by statistical inference

Hence:

$$\delta c = ST$$

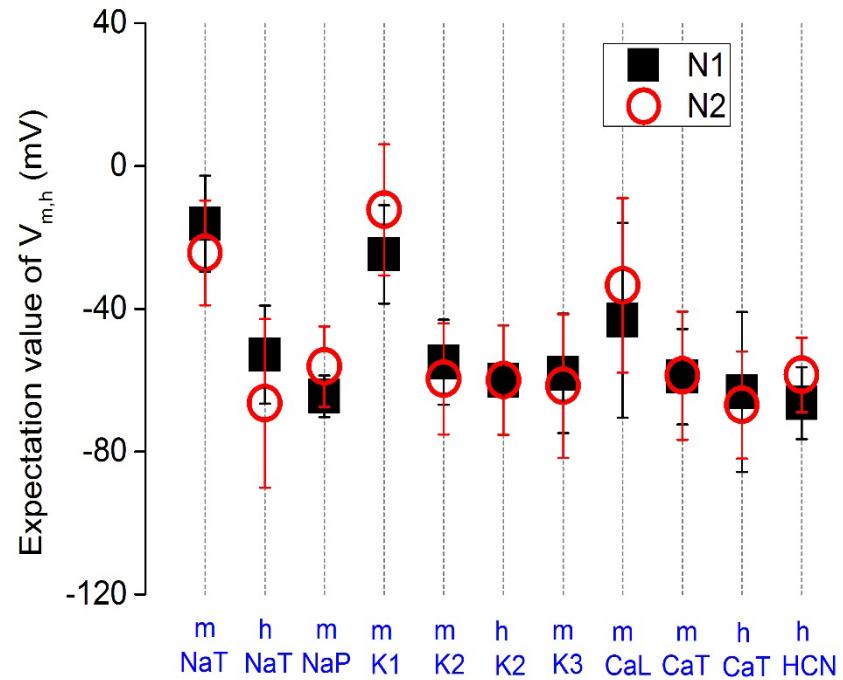
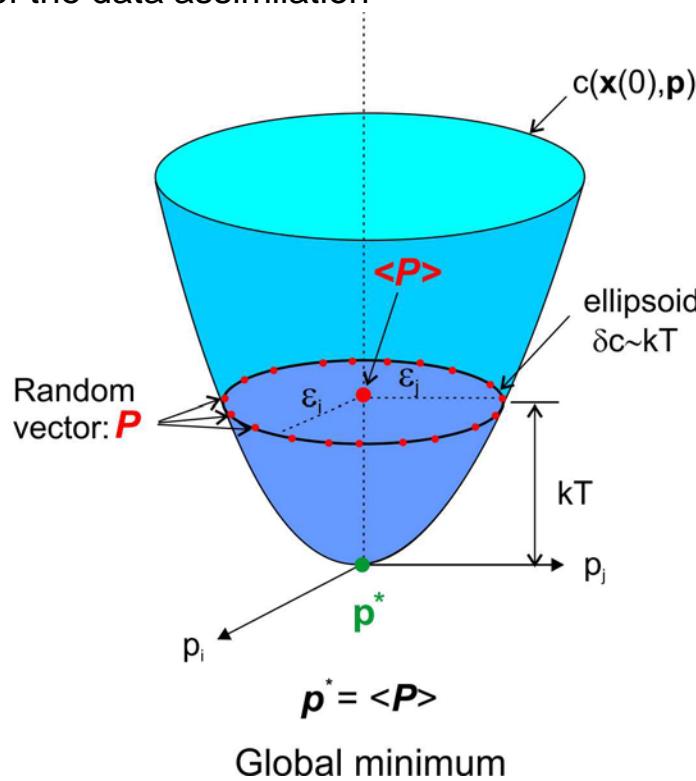
Noise behaves as a residual temperature that prevents reaching the global minimum through direct parameter search

# Statistical inference of the “global” minimum

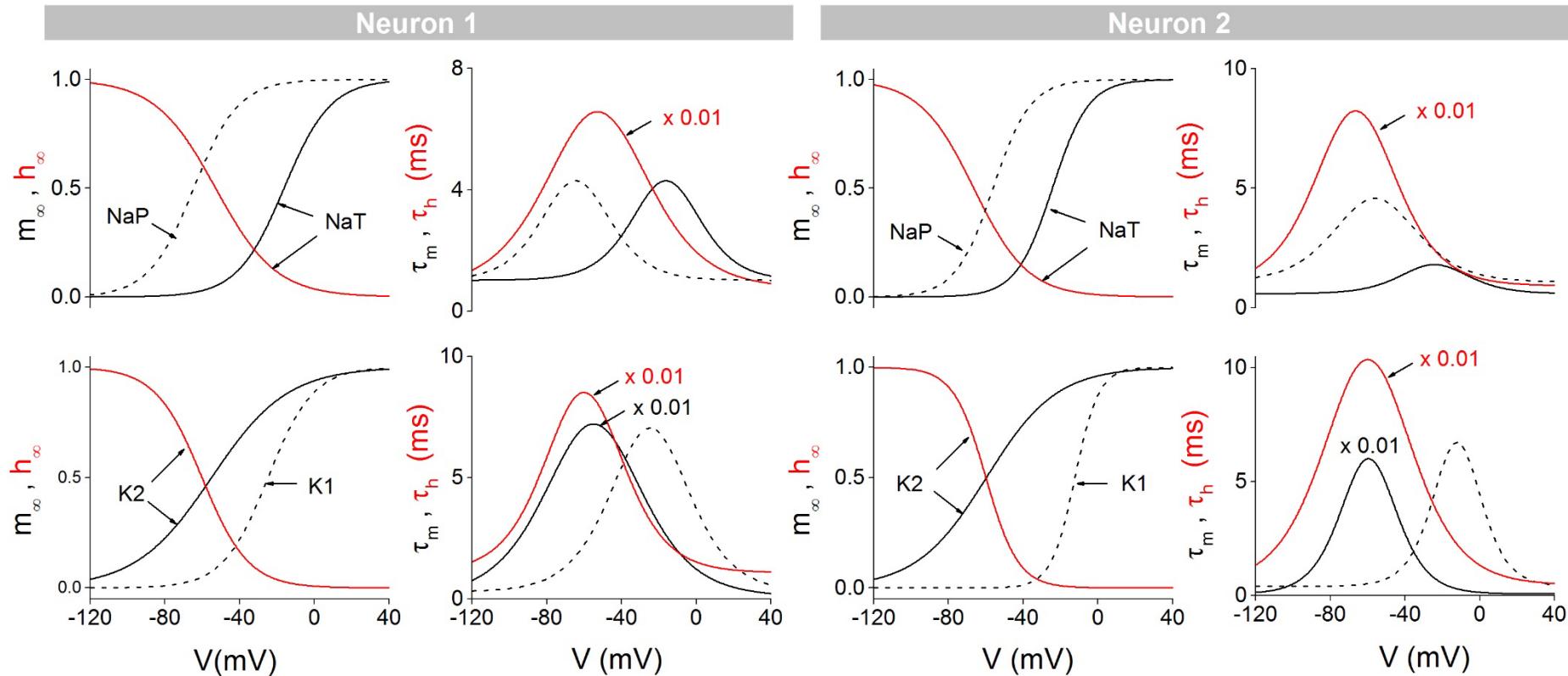
Provided the noise temperature T is not too large that the second order Taylor expansion is accurate:

$$\delta c = \frac{1}{2}(\mathbf{p} - \mathbf{p}^*)^T \hat{\mathbf{H}} (\mathbf{p} - \mathbf{p}^*)$$

the true global minimum  $\mathbf{p}^*$  may be estimated taking a statistical average  $\langle \mathbf{P} \rangle$  of the multivalued solutions of the data assimilation



# Uncertainty on parameters



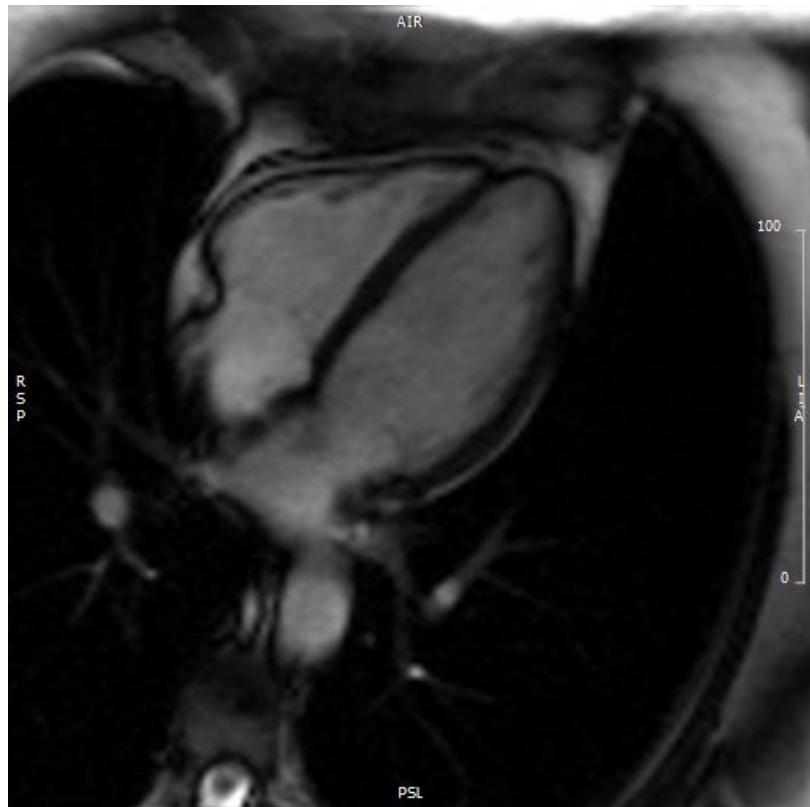
Similarity of the activation curves of 2 HVC neurons reconstructed from the inferred global minimum.

DA + Bayesian inference consistently achieves single valued, biologically plausible models.

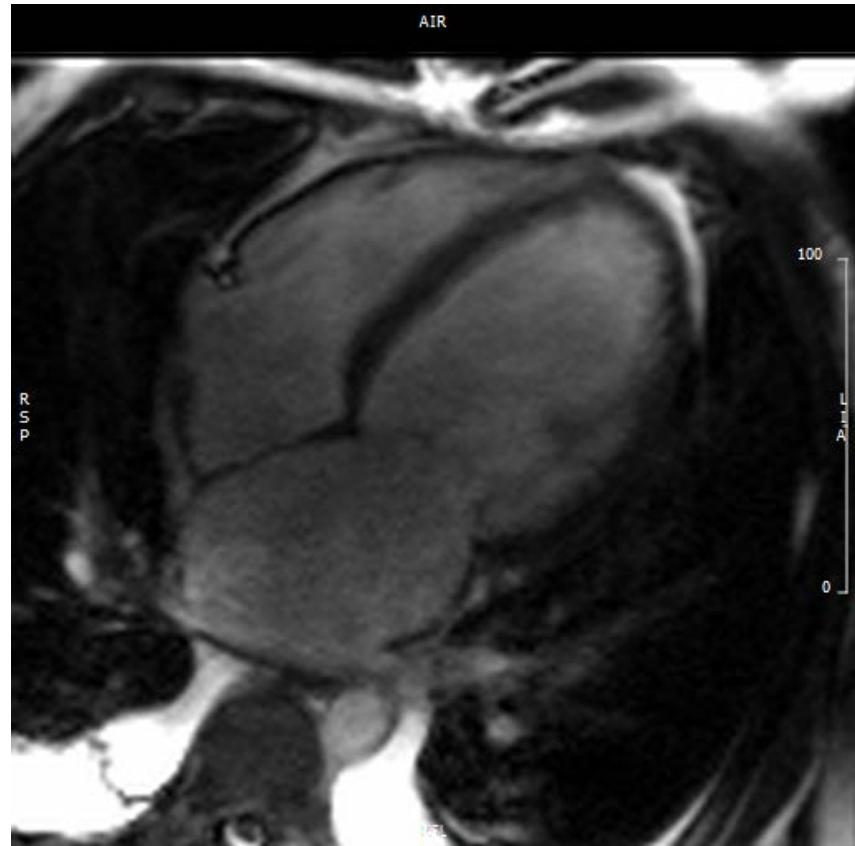
The method is fully automatic, requiring minimal biological intuition.

It absorbs intrinsic fluctuations of biological neurons to make predictions with sufficient accuracy to evidence trial-to-trial fluctuations in neuron behaviour.

# Healthy Heart

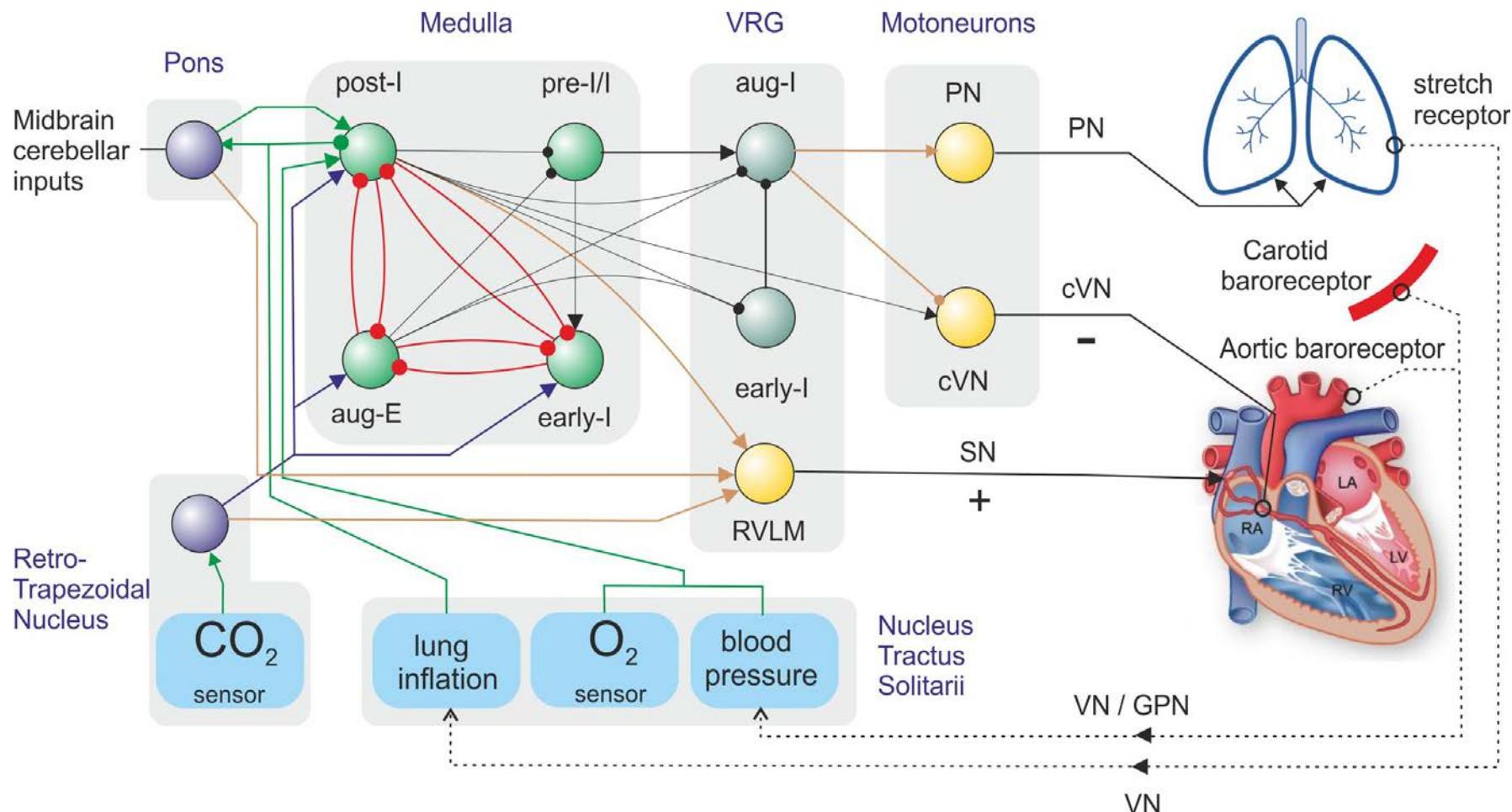


# Heart Failure



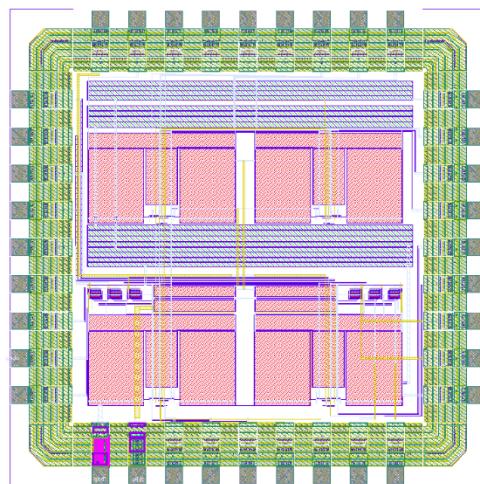
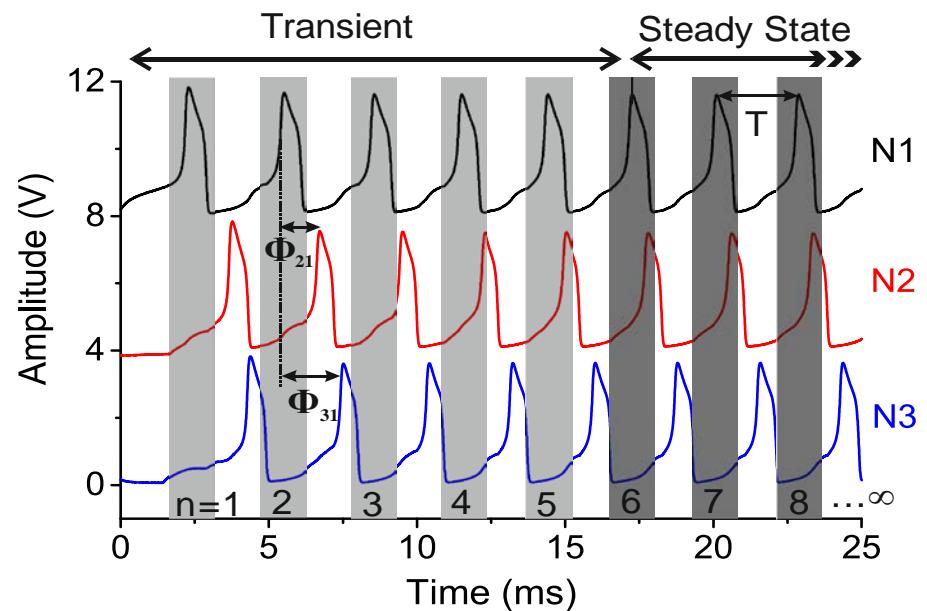
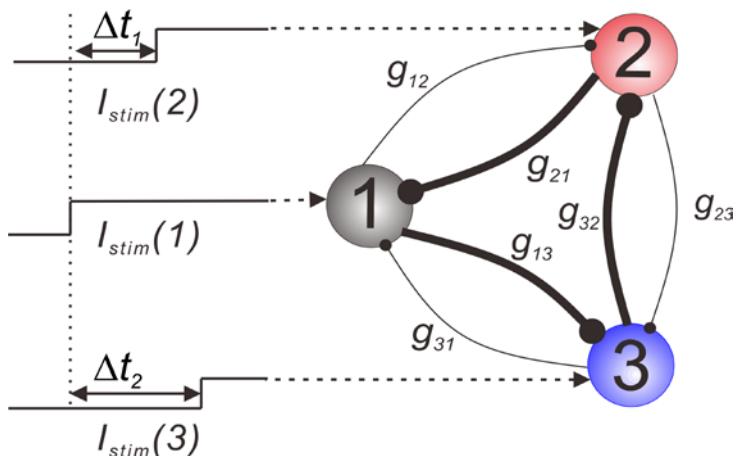
Loss of Heart rate variability is a prognosis of heart-failure

# Medullary Central Pattern Generators control Heart Pacing

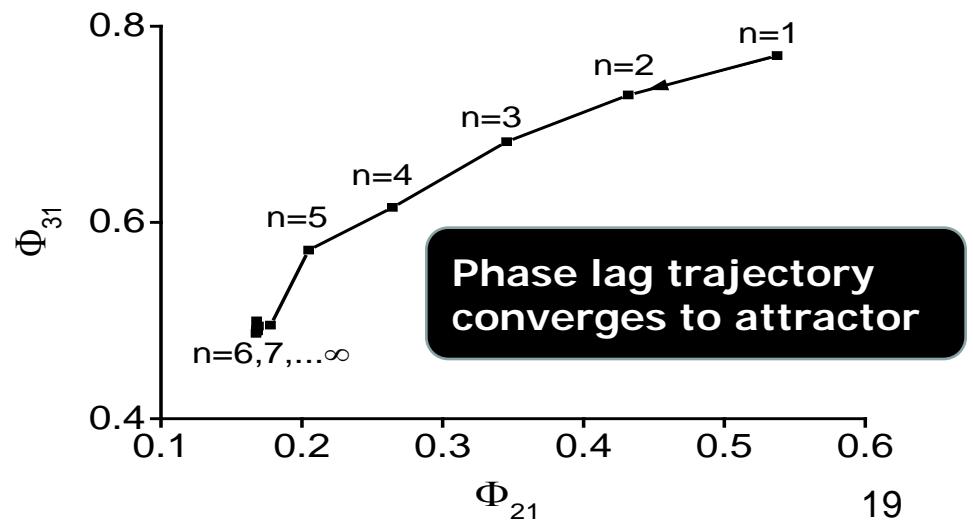


**DA can infer the network connectivity that restores accurate adaptation to physiological feedback**

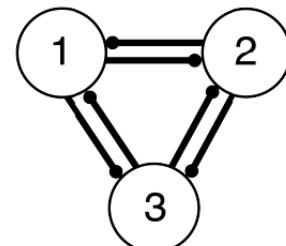
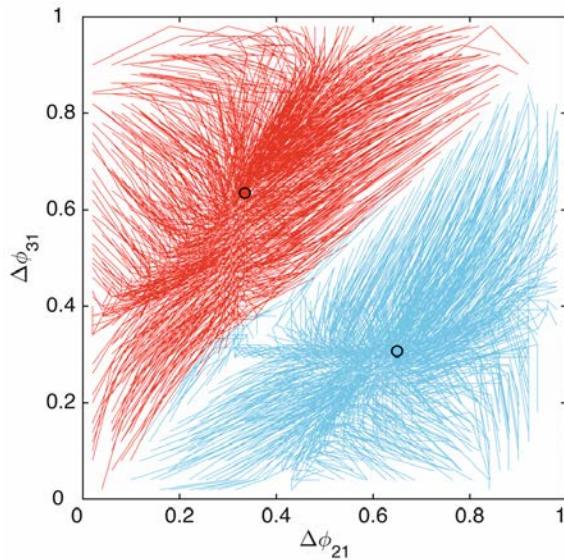
# Application of DA: machine learning



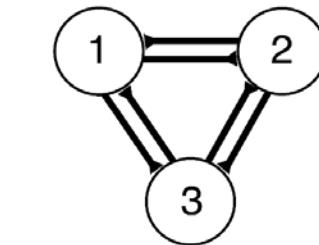
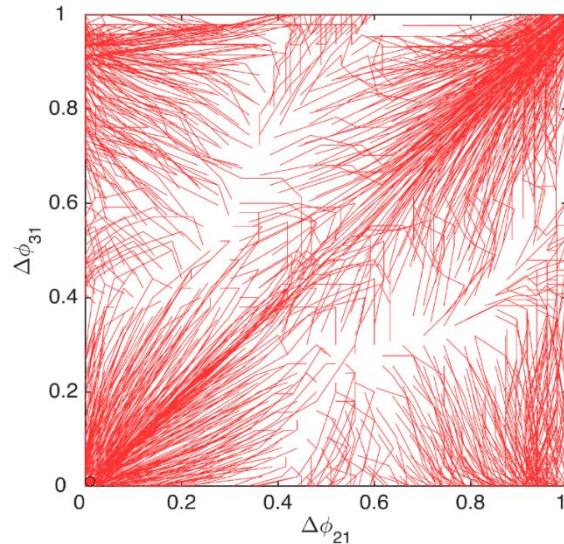
4 HH neuron VLSI platform



## Experiment

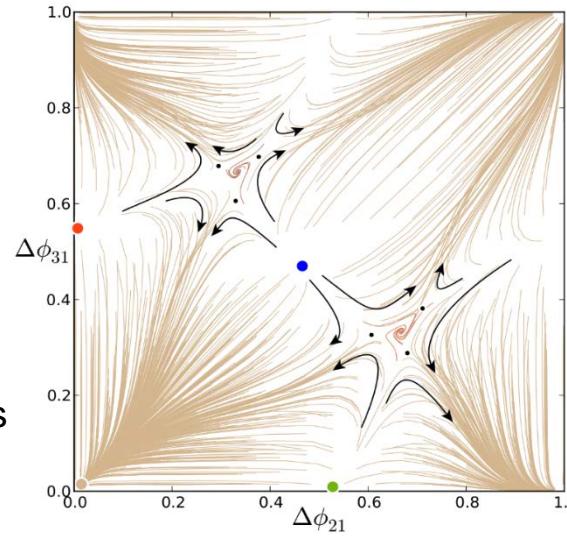
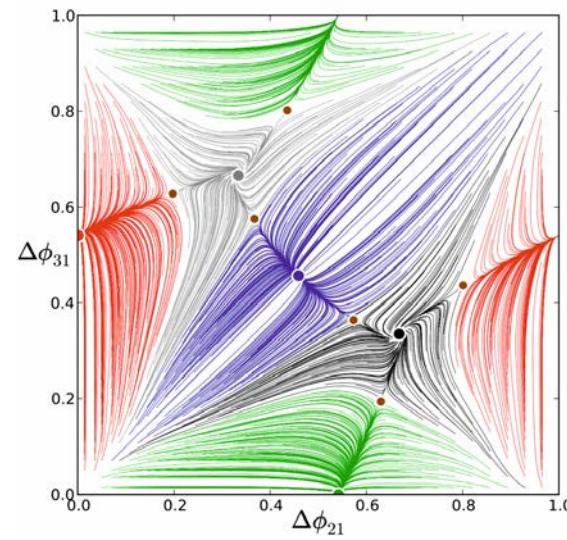


Competing neurons  
(inhibition)



Cooperative neurons  
(excitation)

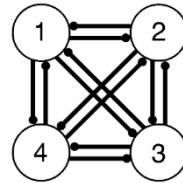
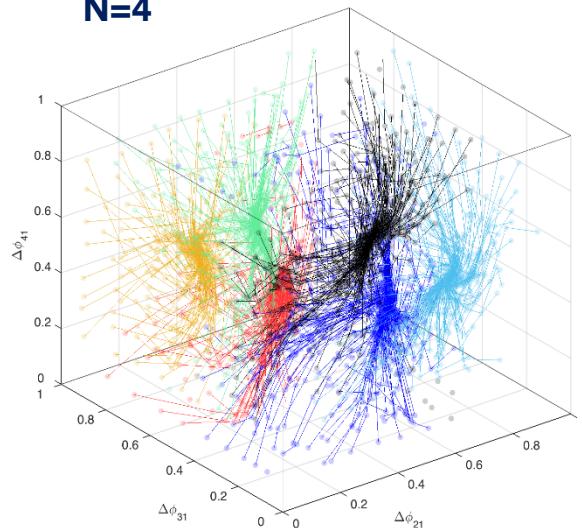
## Theory



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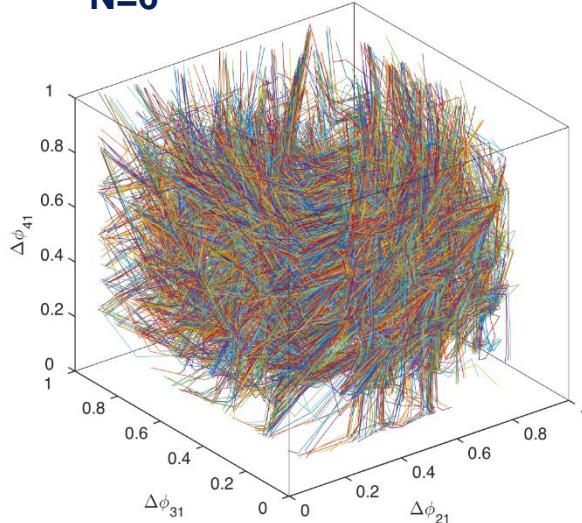
# 4-6 neuron CPGs – electrical switching of motor patterns

**N=4**

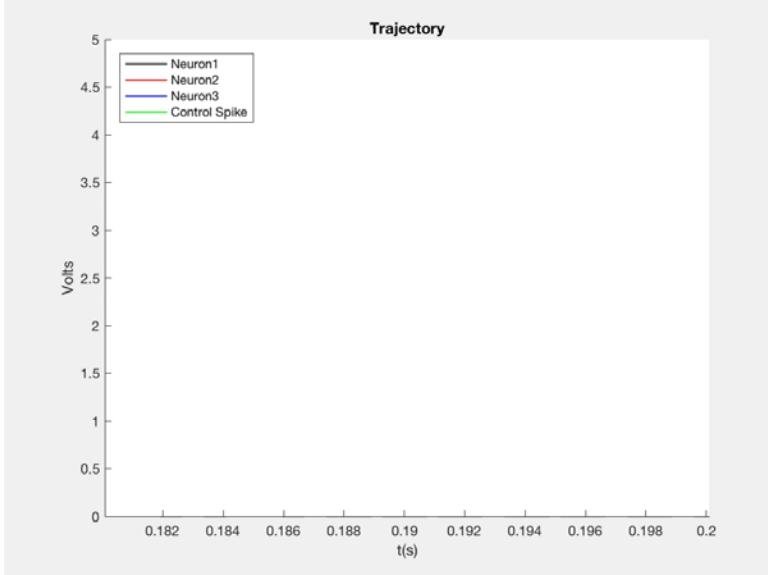


**(N-1)! = 6  
Attractors**

**N=6**



**(N-1)! = 120  
Attractors**

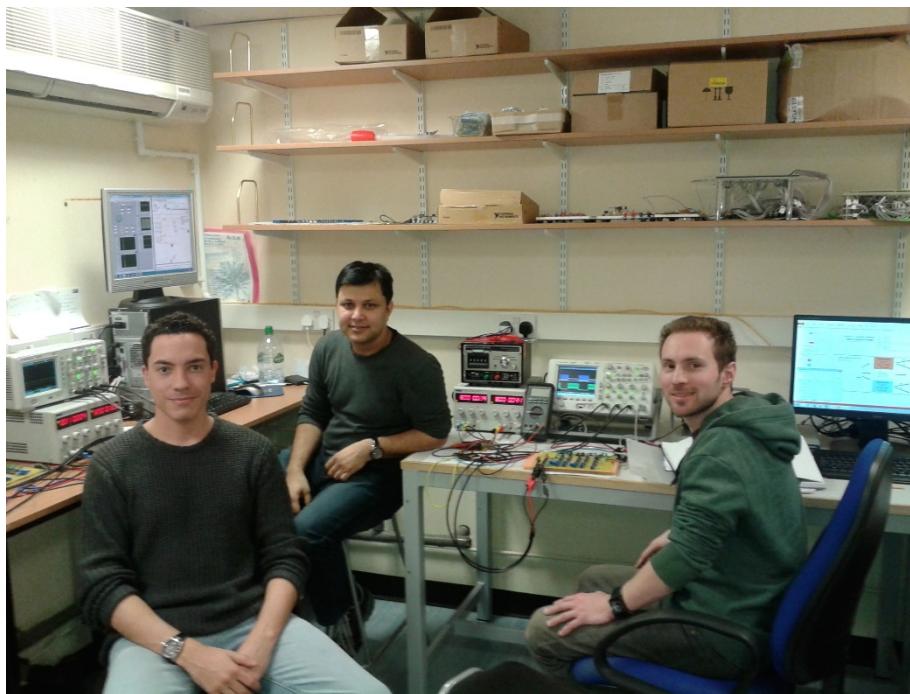


Stimulus induced switching between attractors  
Demonstration of *command neuron action* to switch motor patterns e.g. gaits

- Scaling of attractors  $\sim (N-1)!$
- Switching between attractors is induced by command pulses
- Analog CPGs integrate instantly

DA may “engineer” basins of attraction to produce specific gaits.

# Acknowledgements



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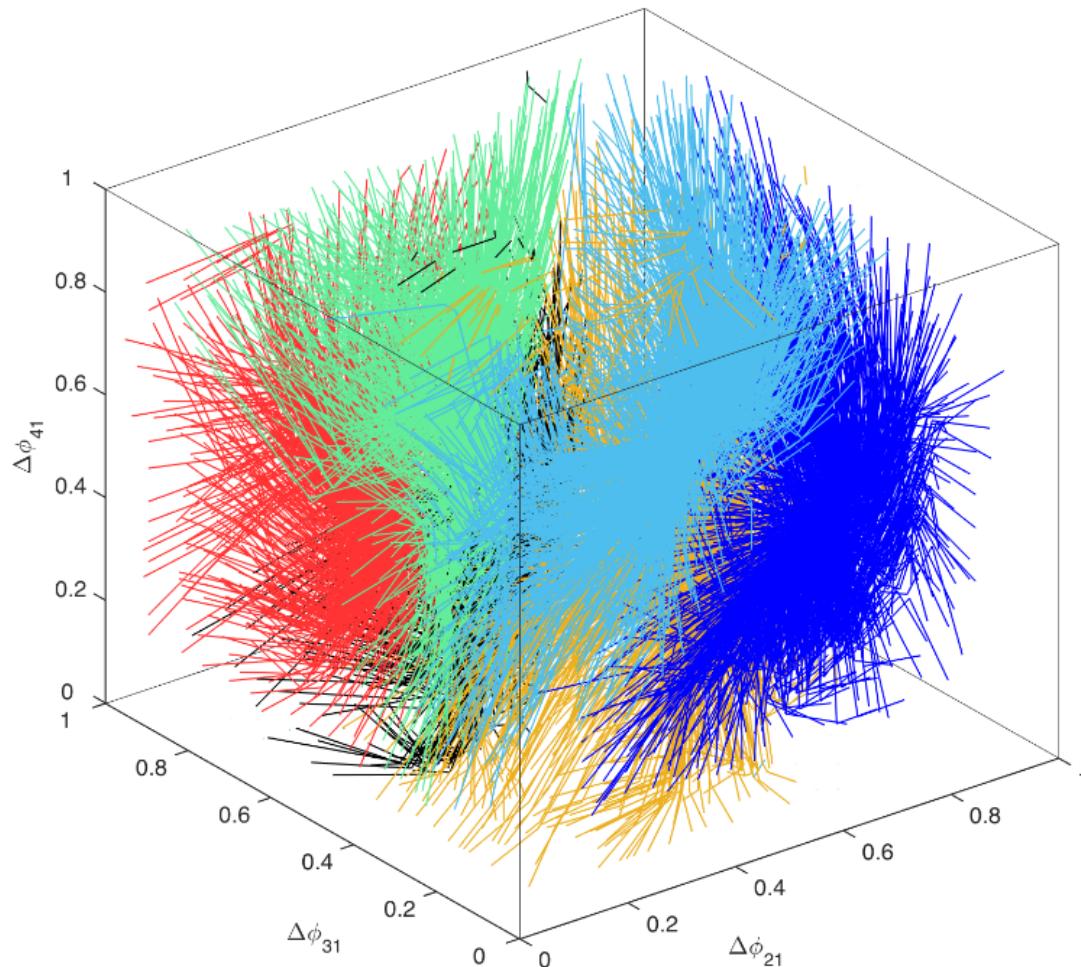
Elizabeth Blackwell Institute for  
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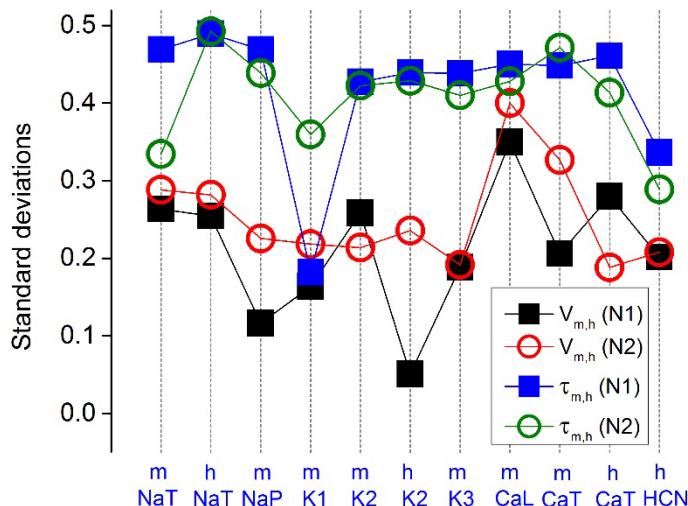
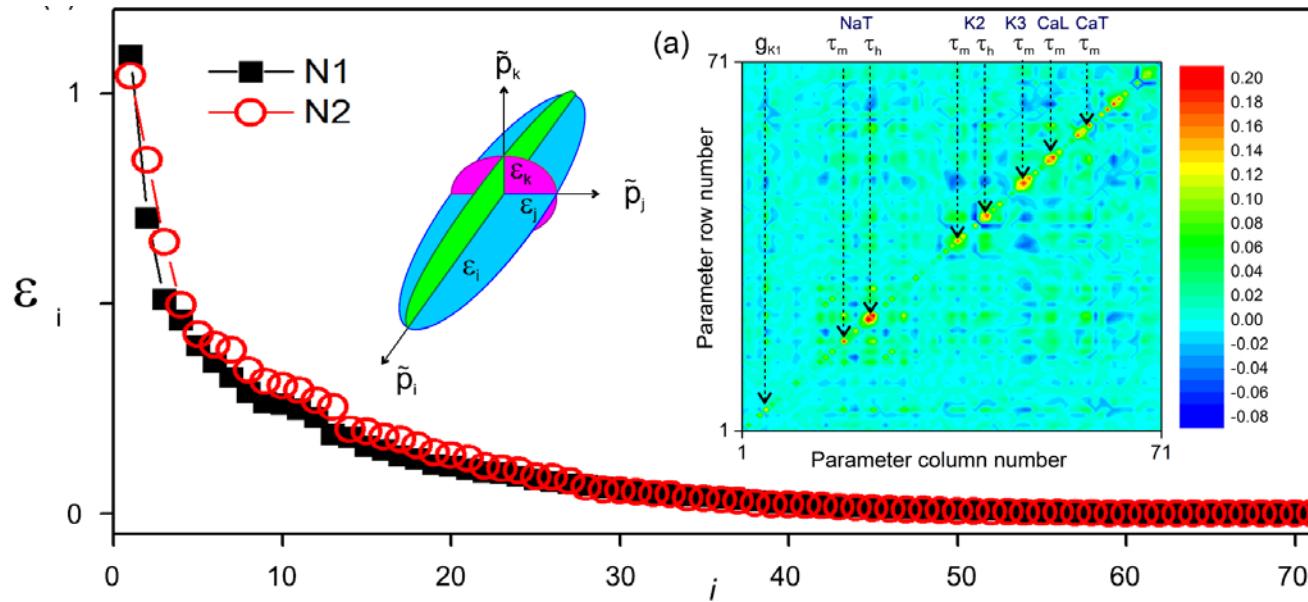


H2020 FET

# Phase map of 3N winnerless network



# Covariance matrix of the data misfit Hessian



The eigenvalues of the Hessian matrix  $\mathbf{H}$  are the half-axes of the 71D-ellipsoid:

The spectrum of eigenvalue decays exponentially.

The “sloppiest” – most loosely constrained – are the time constants of the model, the most tightly constrained are the gate voltage thresholds.